QUESTION 3

Find $\frac{dy}{dx}$ for the following:

$$x^3 + 3xy + y^2 = 15$$

$$x^3 + 3xy + y^2 - 15 = 0$$

$$= \frac{d}{dx} x^3 + \frac{d}{dx} 3xy + \frac{d}{dx} y^2 - \frac{d}{dx} 15 = 0$$

$= \frac{d}{dx} 3xy$, use product rule

let $u = 3x \quad v = y$

$$\frac{d}{d(x)} = 3(xy) = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$= 3x\frac{dy}{dx} + 3y$$

$$= 3x\frac{dy}{dx} + 3y$$

The whole function is:

$$3x^2 + 3x\frac{dy}{dx} + 3y + 2y\frac{dy}{dx} = 0$$

$$3x\frac{dy}{dx} + 2y\frac{dy}{dx} = -3x^2 - 3y$$

$$\frac{dy}{dx}(3x + 2y) = -3x^2 - 3y$$

$$\frac{dy}{dx} = \frac{-3x^2 - 3y^2}{3x + 2y}$$

z.

QUESTION 2

Find the slope of the line tangent to $f(x) = \tan x$ at $x = \frac{\pi}{4}$

$y = mx + b$

$\tan x = \frac{\sin(x)}{\cos(x)}$

$f'(x) = \frac{d}{d(x)}(\tan x)$

$= \frac{d}{d(x)}\left(\frac{\sin(x)}{\cos(x)}\right.$

$= \frac{\frac{d}{d(x)}\sin(x) \cdot \cos(x) - \sin(x)\frac{d}{d(x)}\cos(x)}{\cos^2 x}$

$= \frac{\cos(x) \cdot \cos(x) - \sin(x)(-\sin x)}{\cos^2 x}$

$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$f'(x) = \frac{1}{\cos^2 x}.$

while $x = \frac{\pi}{4}.$

$= \frac{1}{\cos^2(\frac{\pi}{4})} = \frac{1}{0} = \underline{\underline{1}}$

The slope is 1

SPARK 3 Pro

QUESTION 1

Differentiate the following. Simplify your answer completely. Write your answer without negative exponents or rational exponents.

$$f(x) = (3x^4 + 5x - 2)\left[2x^3 + \frac{5}{x^4}\right]$$

$$f'(x) = \frac{d}{dx}\left[3x^4 + 5x - 2\right]\frac{d}{dx}\left[2x^3 + \frac{5}{x^4}\right]$$

$$= \left[\frac{d}{dx}3x^4 + \frac{d}{dx}5x - 2\frac{d}{dx}\right]\left[\frac{d}{dx}2x^3 + \frac{d}{dx}\frac{5}{x^4}\right]$$

$$= (12x^3 + 5 - 0)\left[\left(6x^2 + 5\frac{d}{dx}\left(\frac{1}{x^4}\right)\right]\right.$$

$$= (12x^3 + 5.)\left[6x^2 + 5\frac{d}{dx}(x^{-4})\right]$$

$$= (12x^3 + 5)\left[6x^2 + (-20)x^{-5}\right]$$

$$= (12x^3 + 5)\left(6x^2 - \frac{20}{x^5}\right).$$

$$= \left[72x^5 - \frac{240}{x^2} + 30x^2 - \frac{100}{x^5}\right]$$

$$= \left[72x^5 - 100x^{-5} - 240x^{-2} + 30x^{+2}\right]$$

$$= \left[-28 - 210\right]$$

$$= -238.$$